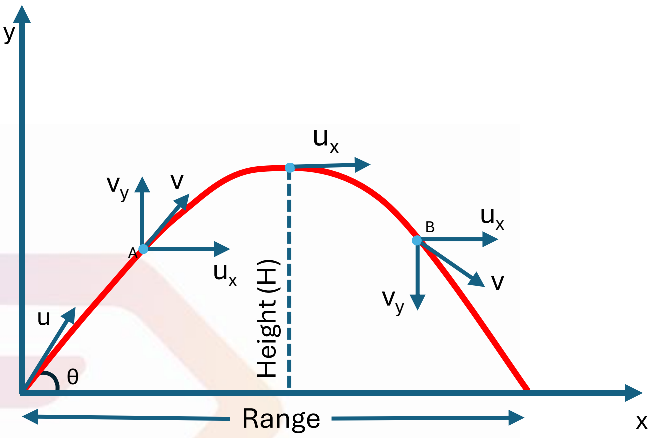
Kinematics

Projectile motion

Derivation of the Equations of Motion

The motion can be broken down into two independent components:



The initial velocity components are:

- **Horizontal component:** $u_x = u\cos(\theta)$
- **Vertical component:** $u_y = u\sin(\theta)$

Horizontal Motion

$$s = ut + \frac{1}{2}at^2$$

For the horizontal motion, the displacement is 'x', initial velocity is u_x, and acceleration 'a' is 0.So, the horizontal distance traveled ('x') at any time 't' is given by:

$$x = (ucos\theta)t$$

$$t = \frac{x}{u\cos\theta}$$

Vertical Motion

In the vertical direction, the only acceleration acting on the projectile is gravity ('g'), which acts downwards. Therefore, the vertical acceleration (ay) is -g.

$$s = ut + \frac{1}{2}at^2$$

For the vertical motion, the displacement is 'y', initial velocity is uy, and acceleration is -g.

$$y = u(sin\theta)t - \frac{1}{2}gt^2$$

$$y = u(sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

This is the **equation of the trajectory of a projectile**, which is the equation of a parabola. This proves that a projectile follows a parabolic path.

Newton's Laws of Motion

Newton's Second Law:

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The direction of the acceleration is in the direction of the net force.

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} = ma$$

Impulse: Impulse is the measure of the overall effect of a force acting over a period of time. It is equal to the change in the momentum of the object that the force acts upon.

$$J = \int F \,\mathrm{d}t = \Delta p$$

Friction: Friction is a force that resists the relative motion or tendency of motion between surfaces in contact.

Static Friction: $f_s \leq \mu_s N$

Kinetic Friction: $f_s = \mu_k N$

Centripetal Force:

A force that acts on a body moving in a circular path and is directed towards the center around which the body is moving. Without this force, the object would travel in a straight line due to inertia.

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Work, Energy ,Power

Work Done:

Work is done on an object when an applied force causes the object to move a certain distance. Importantly, only the component of the force that is in the direction of the object's displacement does work.

$$W = \int F \cdot \mathrm{d}s$$

Work-Energy Theorem:

The net work done on an object by all forces acting on it is equal to the change in the object's kinetic energy.

$$W_{net} = \Delta K = K_f - K_i$$

Kinetic Energy: $K = \frac{1}{2}mv^2 = \frac{P^2}{2m}$

Potential Energy: $U = mgh$ (Gravity)

$U = \frac{1}{2}kx^2$ (Spring)

Conservation of Mechanical Energy

If only conservative forces (like gravity and the elastic force of a spring) act on a system, the total mechanical energy (the sum of kinetic and potential energy) of that system remains constant. Non-conservative forces, such as friction, will cause mechanical energy to be lost (usually as heat).

$$\boldsymbol{K_i + U_i = K_f + U_F}$$

Power: $p_{avg} = \frac{\Delta W}{\Delta t}$ (Average Power)

$P_{int} = F \cdot v$ (Instantaneous Power)

System of Particles & Rotational Motion

Centre of Mass:

The center of mass of an object or a system of particles is the point where the entire mass of the system can be considered to be concentrated for the purpose of analyzing its translational motion. The system moves as if all the external forces were applied at this single point.

$$r_{cm} = \frac{\sum m_i r_i}{\sum m_i}$$

Conservation of Momentum:

The law of conservation of linear momentum states that if the net external force acting on a system is zero, the total linear momentum of the system remains constant.ollision, e =0

Collisions:

Coefficient of restitution:

The coefficient of restitution (e) is a dimensionless number that measures the ratio of the final to initial relative velocity between two objects after they collide. It quantifies how much kinetic energy is conserved in a collision.

For elastic collision, e =1 .

$$\boldsymbol{e} = \frac{\textbf{\textit{Velocity of seperation}}}{\textbf{\textit{Velocity of approach}}}$$

Rotational Kinematics:

$$\omega = \omega_0 + \alpha t \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

System of Particles & Rotational Motion

Torque:

a measure of the turning force that can cause an object to rotate about an axis.

$$\tau = r \times F$$

Moment of Inertia:

Moment of inertia (I) is a measure of an object's resistance to being angularly accelerated. It depends not only on the mass of the object but also on how that mass is distributed relative to the axis of rotation.

$$I = \sum m_i r_i^2$$

Theorems of Moment of Inertia:

Parallel Axis:

This theorem relates the moment of inertia (I) about any axis to the moment of inertia about a parallel axis that passes through the object's center of mass (I_{cm}).

$$I = I_{cm} + Md^2$$

Perpendicular Axis:

This theorem applies to planar objects (thin, flat shapes). It states that the moment of inertia about an axis perpendicular to the plane (I_z) is the sum of the moments of inertia about any two perpendicular axes lying in the plane and intersecting the first axis (I_x and I_y).

$$I_z = I_x + I_y$$

Conservation of Angular Momentum:

This principle states that if the net external torque on a system is zero, its total angular momentum remains constant.